Lebesgue Measure of a set.

Measure of an open interval: The measure of any open interval G is defined as its length and is denoted by the symbol $m(G)$

Measure of Closed interval: Let $[a,b]$ be the smallest closed interval containing a closed set F.

Then we define $m(F) = b-a-m(F')$

F' being the complement of F w.r.t the interval

To prove that the measure of a closed interval is its length.

Let $A = [a,b]$ Then A is a closed interval and hence a closed set $[a,b]$ is the smallest closed interval containing A

Complement of a relative to $[a,b]$ $A' = \phi$ for $A = [a,b]$

By definition $m(A) = b-a-m(A') = b-a-m(\phi) = b-a-0$

$= b-a$

Measure of a Parallelopiped: The volume of an open Parallelopiped

$v(a<x<b,\ c<y<d,\ l<z<m)$ is defined as the measure of v.

Thus $m(v) = (b-a)(d-c)(m-l)$

Similarly the measure of a closed Parallelopiped

$v'(a \le x \le b,\ c \le y \le d,\ l \le z \le m)$ is defined as its volume

Thus $m(v') = (b-a)(d-c)(m-l)$

Exterior and Interior Measure:—